Superconductor-insulator duality for the array of Josephson wires

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We propose novel model system for the studies of superconductor-insulator transitions, which is a regular lattice, whose each link consists of Josephson-junction chain of $N \gg 1$ junctions in sequence. The theory of such an array is developed for the case of semiclassical junctions with the Josephson energy E_J large compared to the junctions's Coulomb energy $E_C = e^2/2C$. Exact duality transformation is derived, which transforms the Hamiltonian of the proposed model into a standard Hamiltonian of JJ array. The nature of the ground state is controlled (in the absence of random offset charges) by the parameter $q \approx N^2 \exp(-\sqrt{8E_J/E_C})$, with superconductive state corresponding to small $q < q_c$. The values of q_c are calculated for magnetic frustrations $f = 0$ and $f = \frac{1}{2}$. Temperature of superconductive transition $T_c(q)$ and $q < q_c$ is estimated for the same values of f. In presence of strong random offset charges, the $T = 0$ phase diagram is controlled by the parameter $\bar{q} = q/\sqrt{N}$; we estimated critical value $\bar{q}_c$.

Introduction and model. Quantum phase transitions (QPT) between superconductive and insulative states in Josephson-junctions (JJ) arrays with submicron-sized junctions were intensively studied, both as function of the ratio between Josephson and charging energies E_J/E_C , and of the applied transverse magnetic field producing frustration of the Josephson couplings (cf. e.g. review [1]). To a large extent, an approach based upon "duality" between Cooper pairs and superconductive vortices [2], was used for theoretical description of phase transition and for interpretation of the data. There are several difficulties related with this approach: i) duality transformation to vortex variables cannot be implemented exactly for the standard Hamiltonian of JJ array, and some poorly controlled approximations are necessarily used, ii) comparison of theory with experiments is complicated by the fact that the normal-state resistance of junctions R_n is close to quantum resistance $R_Q = h/4e^2$ in the transition region, thus $E_J \sim E_C \sim \Delta$ and standard approximation of the local in time, "phase-only" Hamltonian cannot be justified, iii) randomly frozen "off-set" charges known to exist in all JJ arrays introduce random frustration into the kinetic energy term for vortices; the role and relative importance of this effect for the S-I transition is barely unknown.

In the present Letter we propose and study modified version of JJ array (shown in Fig.1) which possesses quantum phase transition within parameter range $E_J \gg E_C = e^2/2C$. Each single bond of this novel array contains a chain (refered to as Josephson wire, JW) of $N \gg 1$ identical junctions with Josephson energy E_J and capacitance C. We neglect self-capacitances C_{isl} of islands compared to junctions capacitances C. Lagrangian of this array ($\mathcal{M} \times \mathcal{M}$ plackets) is:

$$\mathcal{L} = \sum_j \left[\frac{1}{16E_C} \left(\frac{d\vartheta_j}{dt} \right)^2 + E_J \cos \vartheta_j \right]. \qquad (1)$$

where sum goes over all junctions shown in Fig.1, and ϑ_j is the phase difference on the j-th junction. Phase

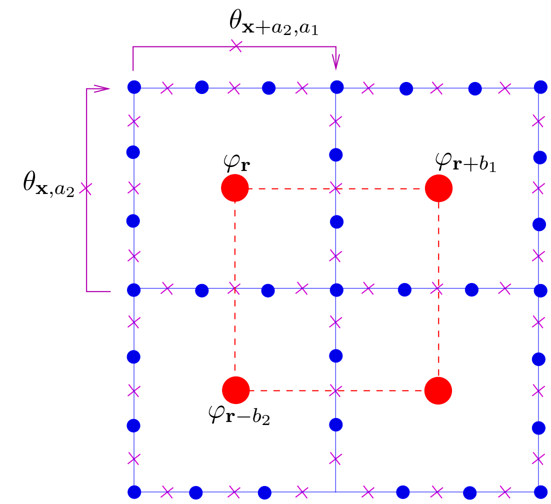

Figure 1: Fig. 1. The array of Josephson wires. Small circles represent the superconducting islands connected by Josephson junctions (crosses). The phase differences $\theta_{\mathbf{x},a_\mu}$ are defined on the bonds of the array. The large circles denote the vertices of the dual lattice.

differences ϑ_j are subject to the constraints on each lattice placket (counted by dual lattice coordinate $\mathbf{r}$): $\sum_\square \vartheta_j = 2\pi f_\mathbf{r} = 2\pi\Phi_\mathbf{r}/\Phi_0$, where $\Phi_\mathbf{r}$ is external magnetic flux through the placket $\mathbf{r}$. An effective Josephson coupling E_J^{eff} between the nodes of JJ lattice is suppressed as E_J/N, whereas effective amplitude of quantum phase slip processes (i.e. amplitude of vortex tunnelling) is enhanced, either $\propto N$ in the absence of off-set charges, or $\propto \sqrt{N}$, if off-set charge disorder is strong. Therefore, at sufficiently large N the whole array will become insulating even if the ratio E_J/E_C is large. Such a model possess two important features which makes theoretical analysis simpler: i) for a long chain of junctions, semiclassical energy-phase relation $E(\phi)$ is piecewise parabolic, with a period $\phi \in (-\pi, \pi)$, and ii) an amplitude v of an individual quantum phase slip in each

of N junctions is small, $v \ll E_J^{\rm eff}$; therefore the simplest vortex tunnelling Hamiltonian is an adequate description of multiple phase slips. On experimental side, the advantages of the proposed system are: i) an effective Josephson frequency of an array can be made small, allowing for clear separation between collective bosonic excitations of an array and single-electron excitations within superconductive islands, and ii) superconductor-insulator transition can be explored with a set of arrays with exactly same parameters E_J and E_C as function of N.

Duality transformation. Following paper [3] where ground-state quantum properties of a single Josephson wire was analysed, we present classical Josephson energy of our array in presence of frustrating magnetic field in the form:

$$E_{cl} = \frac{E_J}{2N} \sum_{r,\,\mu} \left(\theta_{\mathbf{x},\,a_\mu} - 2\pi q_{\mathbf{x},\,a_\mu}\right)^2 \,, a_1 = (1,0)\,, a_2 = (0,1) \tag{2}$$

where $q_{\mathbf{x},\,a_\mu}$ are integer numbers, $\theta_{\mathbf{x},\,a_\mu}$ are phase variables associated with bonds of the lattice and subject to the set of constraints $\theta_{\mathbf{x},\,a_2} + \theta_{\mathbf{x}+a_2,\,a_1} - \theta_{\mathbf{x},\,a_1} - \theta_{\mathbf{x}+a_1,\,a_2} = 2\pi f_{\mathbf{r}}$. Minimization over θ's in presence of constraints lead to an equivalent expression in terms of vortex variables p_r:

$$E_{cl} = \frac{2\pi^2 E_J}{N} \sum_{r,\,r'} G_{r,\,r'} \left(p_r - f_r\right)\left(p_{r'} - f_{r'}\right) \tag{3}$$

where $G_{r,\,r'}$ is the Green function of Laplacian operator on a square lattice; in Fourier space $G^{-1} = 4 - 2\cos\kappa_x - 2\cos\kappa_y$. Note that the same Green function determines Coulomb interaction between Cooper pairs located at the node islands $\mathbf{x}$ and $\mathbf{x}'$ of the original lattice: $E_C(\mathbf{x},\mathbf{x}') = N\frac{(2e)^2}{C} G_{\mathbf{x},\,\mathbf{x}'}$.

To construct quantum Hamiltonian in vortex variables, we introduce a set of "second-quantized" operators $a_{\{p\}}$ and $a^+_{\{p\}}$ (a pair of operators for each set of vorticities $\{p\}$). Classical states can be viewed as an infinite-dimensional lattice (with the dimensionality equal to the number of sites of the lattice dual to the original JJ array). A quantum phase slip in a junction is the process which changes the vorticities in two neighboring cells by ± 1, with an amplitude $\Upsilon_{p,p'}$. The Hamiltonian then reads

$$H = \sum_{\{p\}} E_{cl}\left(\{p\}\right) a^+_{\{p\}} a_{\{p\}} - \frac{1}{2} \sum_{\langle \{p\}\,,\{p'\}\rangle} \Upsilon_{p,p'} a^+_{\{p\}} a_{\{p'\}} \tag{4}$$

The first sum runs over all the configurations of the vortices. The second sum runs over all nearest-neighbors *directed* bonds in the lattice of the classical states of the array. By definition the nearest-neighbors in this lattice are the sites connected by one quantum phase slip, therefore configurations p and p' differ by their vorticities in two neighbouring dual sites r and $r' = r + b$. It is possible (due to neglect of C_{ins}) to show that tunnelling amplitudes $\Upsilon_{p,p'}$ depend on the dual coordinates r, r' only, i.e. it does not depend on all other parameters specifying configurations p and p'. Below we denote this amplitude as $\Upsilon_{r,r'}$ and will specify its explicit form later. The next step is to perform Fourier transformation from the set of integers $\{p\}$ into the set of phase variables φ_r associated with sites of dual lattice, according to $a_{\{\varphi\}} = \sum_{\{p\}} a_{\{p\}} e^{ip\varphi}\,, \quad a_{\{p\}} = \int D\varphi\; a_{\{\varphi\}} e^{-ip\varphi}$. Now the Hamiltonian (4) can be written as

$$H = \prod_r d\varphi_r \left[\frac{(2\pi)^2 E_J}{2N} a^+_{\{\varphi\}} \widehat{\mathcal{L}} a_{\{\varphi\}} \right.$$
$$\left. - \frac{1}{2} a^+_{\{\varphi\}} a_{\{\varphi\}} \sum_{[\vec{r}\,,\vec{r}']} \left[\Upsilon_{r,r'} \exp\left(i\varphi_{\vec{r}} - i\varphi_{\vec{r}'}\right) + h.c.\right] \right] \tag{5}$$

where $\widehat{\mathcal{L}} = \sum_{\vec{r},\vec{r}'} G_{\vec{r},\vec{r}'} \left(-i\frac{\partial}{\partial\varphi_{\vec{r}}} - f\right)\left(-i\frac{\partial}{\partial\varphi_{\vec{r}'}} - f\right)$ and the sum is taken over all non-directed bonds on the dual lattice. The corresponding first-quantized "dual" Hamiltonian in terms of vortex number and phase operators $\hat{N}_r$ and φ_r reads:

$$H^{\rm dual} = 4\tilde{E}_C \sum_{r,r} (\hat{N}_r - f) G_{r,r'} (\hat{N}_{r'} - f) \; -$$
$$- \frac{1}{2} \sum_{[\vec{r}\,,\vec{r}']} \left[|\Upsilon_{r,r'}| e^{i\left(\varphi_{\vec{r}} - \varphi_{\vec{r}'} + \chi_{r,r'}\right)} + \text{H.c.} \right] \tag{6}$$

where $\chi_{r,r'} = \text{Arg}\Upsilon_{r,r'}$. In Eq. (6) we define "dual charging energy"

$$\tilde{E}_C = \pi^2 E_J / 2N, \tag{7}$$

uniform "charge" frustration $f \in (0,1)$, frustrated "dual Josephson" couplings with the local strengths $\tilde{E}_J(r,r') = |\Upsilon_{r,r'}|$ and "magnetic" frustration parameters $\Gamma_x = \frac{1}{2\pi}\sum_{\square} \chi_{r,r'}$. The Hamiltonian (6) is of the standard form for the Josephson-junction array with junction-dominated capacitive energy. An important remark is in order: the Hamiltonian defined by (6) was derived neglecting single-electron excitations within each superconducting island; this is legitimate below the parity effect temperature $T^* \approx \Delta/\log(\nu\Delta V)$ only [4]; we will assume $T < T^*$ below.

If the original array is free from background charges, one finds, following derivation in Ref.[3], that $\Upsilon_{r,r'} \equiv \Upsilon \gamma^{(1)}_{r,r'}$, where $\Upsilon = 2Nv$, and $\gamma^{(1)}_{r,r'} = 1$ for nearest neigbouring cites r , r' on the dual square lattice, and zero otherwise.

$$v = \frac{2^{11/4}}{\sqrt{\pi}} \left(E_J^3 E_C\right)^{1/4} \exp\left[-2\sqrt{\frac{2E_J}{E_C}}\right] \tag{8}$$

is the amplitude of a tunneling process (quantum phase slip) in each of N junctions which constitute an elementary link of the JJ array. In this case $\tilde{E}_J = \Upsilon = 2Nv$ whereas $\Gamma_r \equiv 0$. The nature of the ground state is then controlled by the value of

$$q = \tilde{E}_J/\tilde{E}_C = 4N^2 v/\pi^2 E_J. \tag{9}$$

The "insulative" (in dual variables) state is realized (at $f = 0$) for $q < q_c \approx 0.5$, according to the lowest-order variational calculation [5] and Quantum Monte-Carlo simulations [6, 7]. Below we extend the calculation of ref. [5] and find q_c for $\frac{1}{2}$; we will also find $T = 0$ expression for the superconducting density $\rho_s(q)$ of the wire array at $q < q_c$. Insulating state of the wire array is realized at $q > q_c$; here we calculate effective dielectric permeability $\varepsilon(q)$.

Background off-set charges coupled to the "bond" islands modify [8] *phases* of amplitudes of phase slips in different junctions: $\upsilon_k = \upsilon e^{i\chi_k}$. If off-set charge disorder is strong, phases χ_k are totally random and distributed over the circle $(0, 2\pi)$. As a result, tunnelling amplitudes $\Upsilon_{r,r'}$ constitute now Hermitian random matrix with Gaussian statistics:

$$\Upsilon_{r,r'} = \tilde{E}_J^d \cdot \gamma^{(1)}_{r,r'} \cdot z_{r,r'}, \quad \tilde{E}_J^d = 2\sqrt{N}\upsilon, \quad \overline{|z_{r,r'}|^2} = 1. \tag{10}$$

The strength of "dual Josephson coupling" is suppressed due to charge disorder by the factor $1/\sqrt{N}$, and relevant control parameter is now $\overline{q} = q/\sqrt{N}$. Moreover, dual array with couplings $\Upsilon_{r,r'}$ is randomly frustrated due to randomness of phases $\chi_{r,r'} = \mathrm{Arg}\Upsilon_{r,r'}$. Critical value $\tilde{q}_c$ for the Hamiltonian (6) with random matrix $\Upsilon_{r,r'}$ will be calculated below.

Off-set charges Q_x related to the "node" islands contribute directly to the frustration parameter Γ_x

$$\Gamma_x = Q_x + \Gamma_x(Q_x = 0) \tag{11}$$

Eq. (11) is useful for derivation of the relation (13) below.

To complete duality transformation, we need to identify dual partners for the superconducting density ρ_s and dielectric permeability ε characterising electromagnetic response of an original array. Superconducting density is defined via the energy $E_s\{\theta\} = \frac{\rho_s}{2}\int d^2x(\nabla\theta)^2$ of an inhomogeneous state, it is related to kinetic inductance per square: $\mathcal{L}^K_\square = \Phi_0^2/(4\pi^2\rho_s)$. We calculate ρ_s by introducing infinitesimal vector potential $\delta\mathbf{A}$ modifying magnetic frustration of the original array, which transforms into modification of "charge frustration" f_r in the dual representation. Dielectric permeability ε of the original array is calculated via an energy response to the introduction of an additional infinitesimal stray charges $Q_{\mathbf{x}} = \delta Q$ and $Q_{\mathbf{x}'} = -\delta Q$ via 2D Coulomb relation $d^2E/d(\delta Q)^2 = [N/\varepsilon C]\, G_{\mathbf{x},\mathbf{x}'}$. Variation of stray charges transform then into a variation of "magnetic" frustration in the dual representation. Simple calculations lead to the dual relations

$$\rho_s = \frac{E_J}{N} \cdot \varepsilon_D^{-1} \tag{12}$$

$$\varepsilon^{-1} = \frac{\pi^2}{2NE_C}\rho_s^D \tag{13}$$

ε_D is the effective dielectric permeability of *dual* array and ρ_s^D is its effective superconducting density, which are defined in the insulating and superconducting states of the dual array, correspondingly.

S-I transition point at $T = 0$: variational method.

Variational method for the Hamiltonian $H = H_C + H_J$ defined by Eq.(6) was developed in [5] for the determination of the transition point. The idea of this method (we use it at $T = 0$ and for static case only) is to consider the ground-state energy E_{var} as a bilinear functional of average values $\psi_i = \langle e^{i\phi_i}\rangle$, i.e. $E_{var} = \sum_{r_1,r_2} L_{r_1,r_2}\psi^*_{r_1}\psi_{r_2}$, and to determine the condition for the operator $\hat{L}$ to acquire zero mode. This calculation was performed in [5] for $f = 0$. We generalized such a calculation for the case $f = \frac{1}{2}$ as well; the matrix L_{r_1,r_2} is presented below:

$$L_{r_1r_2} = \epsilon_+\left(\delta_{r_1r_2} - c_f\frac{\tilde{E}_J}{\epsilon_1}\gamma^{(1)}_{r_1r_2}\right) \tag{14}$$

where $\epsilon_1 = 2\tilde{E}_C$ is the Coulomb energy of the smallest $(+,-)$ dipole residing on nearest-neighbouring sites, and $\epsilon_+ = (2\tilde{E}_C/\pi)\log\mathcal{M}$ is the Coulomb energy of a single-charge excitation, $c_0 = 1$ and $c_{\frac{1}{2}} = \frac{4}{3}$. Eq.(14) is derived in the main approximation over small parameter $\epsilon_1/\epsilon_+ \sim \log^{-1}\mathcal{M}$. The result for criticial values of $q = \tilde{E}_J/\tilde{E}_C$ reads:

$$q_c = \frac{1}{2} \text{ for } f = 0 \qquad q_c = \frac{3}{8} \text{ for } f = \frac{1}{2} \tag{15}$$

Superconducting density ρ_s and phase diagram without off-set charges.

At $q < q_c$ and low temperatures $T < T_{sup}(q)$ the Josephson array is superconductive. Superconductive density ρ_s coincides with E_J/N in the absence of both thermal and quantum fluctuations, $T \to 0$ and $q \to 0$. We start from analysing quantum corrections to ρ_s at $T = 0$, making use of the dual relation (12). The ground state of the dual array with the Hamiltonian (6) is insulating, its dielectric permeability ε_D can be expressed in terms of the Fourrier-transform $R(p, 0)$ of the irreducible zero (Matsubara) frequency charge-charge correlation function $R(r, \omega = 0) = \int d\tau\langle\langle N_r(\tau)N_0(0)\rangle\rangle$:

$$\frac{1}{\varepsilon_D} = 1 - 8E_C\frac{R(p,0)}{p^2} \tag{16}$$

Correlation function $R(p, 0)$ can be expanded in series over "dual Josephson" part of the Hamiltonian (6), this explanation contains even powers of $q = \tilde{E}_J/\tilde{E}_C$ only. We calculated $R(p, 0)$ for the $f = 0$ case up to the 4-th order in q. Details of this rather tedious calculations will be presented elsewhere [9], the result is

$$\rho_s = \frac{E_J}{N}\left[1 - q^2 - (a_{\mathrm{p}} + a_{\mathrm{r}})q^4\right], \quad a_{\mathrm{p}} = 0.84 \quad a_{\mathrm{r}} = 2.42 \tag{17}$$

Here coefficient a_{p} corresponds to the contribution of diagrams which include two couplings Υ_{r_1,r_2} and Υ_{r_3,r_4} with pair-wise equal coordinates $r_1 = r_4$ and $r_2 = r_3$, whereas coefficient a_{r} corresponds to "ring" diagrams

with all four different points $r_{1,2,3,4}$ (all diagrams contributing in the order q^2 contain products $|\Upsilon_{r,r'}|^2$ only). The result (17) is reliable as long as 4th-order correction is small compared to the 2nd-order one, i.e. $q \le 0.4$. Eqs. (12) and (17) determines reduction of the $T = 0$ superconducting density due to quantum fluctuations of vortices beyond vortex-free ground-state. Upon temperature increase, superconductivity is destroyed according to Berezinsky-Kosterlitz-Thouless mechanism of vortex depairing, with transition temperature $T_{\rm BKT} = A\frac{\pi}{2}\rho_s(T = 0)$. Suppression factor $A_0 = 0.87$ was found numerically [10, 11] for classical phase transition in the Gaussian periodic XY model like the one we study here, for $f = 0$. In the fully frustrated case $f = \frac{1}{2}$ suppression is stronger [11, 12], $A_{\frac{1}{2}} = 0.52$. Full line in Fig.2 presents q-dependence of the superconducting transition temperature $T_{\rm sup}(q) = \pi A_0 E_J/2N\varepsilon_D$.

In presence of magnetic frustration $f \neq 0$ calculations of quantum corrections to ε_D up the 4-th order in $\tilde{E}_J$ looks complicated, here we present 2-nd order results only:

$$\rho_s = \frac{E_J}{N}\left(1 - \frac{112}{27}q^2\right). \tag{18}$$

The corresponding superconducting transition temperature $T_{\rm sup} = \pi A_{\frac{1}{2}} E_J/2N\varepsilon_D$ as function of q is shown in Fig. 2 by the line with crosses.

At $q > q_c$ the ground state of the dual Hamiltonian contains Bose-condensed vortices. Very deep inside the dual superfluid state ($q \gg q_c$) the corresponding "dual superfluid density" $\rho_s^D(q = \infty) = \tilde{E}_J = 2N\upsilon$, cf. last term of the Hamiltonian (6). Such a state possesses collective excitation with frequency

$$\omega_J^D = \sqrt{8\tilde{E}_C\tilde{E}_J} = 2^{3/2}\pi\sqrt{\upsilon E_J}, \tag{19}$$

which is a dual analog of usual Josephson plasma oscillations with much higher frequency $\omega_J = \sqrt{8E_JE_C} \gg \omega_J^D$. Finite-$q$ correction to ρ_s^D in the lowest order over q^{-1} is due to anharmonicity of the zero-point fluctuations of phases φ_r; it can be calculated as $\rho_s^D = \rho_s^D(q = \infty)\langle\cos(\varphi_r - \varphi_{r+b})\rangle = \rho_s^D(1 - 1/\sqrt{8q})$. Note that perturbative corrections to ρ_s^D do not depend on density of "dual charges" controlled by f.

According to Eqs.(8) and (13), the original array is then in the insulating ground state with inverse dielectric permeability

$$\varepsilon^{-1} = 2^{11/4}\pi^{3/2}\left(\frac{E_J}{E_C}\right)^{3/4} e^{-\sqrt{8E_J/E_C}}\left(1 - \frac{1}{\sqrt{8q}}\right) \tag{20}$$

Interaction of $2e$ charges in such an array is logarithmic, $U(x) = (4NE_C/\pi\varepsilon)\log(x)$, the corresponding BKT charge unbinding temperature is $T_{\rm ins} = 0.57NE_C/\pi\varepsilon$ [1]. Note that dielectric constant ε is very large in the whole range of applicability of our theory; this is due to our major assumption of $E_J \gg E_C$. The line with asterisks marks on Fig. 2 shows the normalized transition

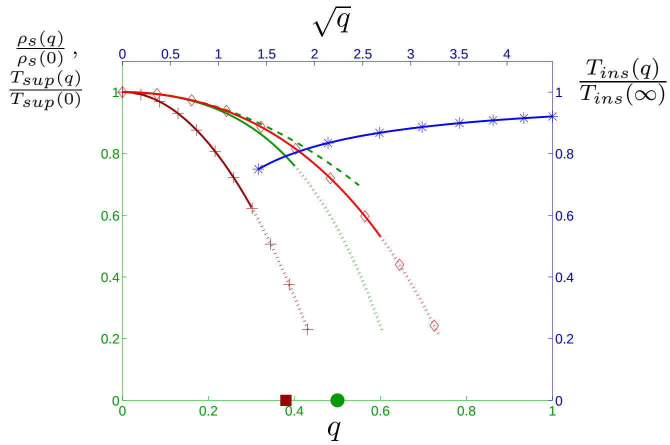

Figure 2: Fig. 2. Temperatures of various phase transitions in the Josephson wire array. The full line with asterisks shows the normalized metal-insulator transition temperature (*right* axis) versus $\sqrt{q} = \sqrt{\tilde{E}_J/\tilde{E}_C}$ (*top* axis) in the limit of large q and in the absense of random stray charges. All the other lines should be referred to the *bottom* and *left* axes. They show the normalized superfluid density and superconductor-normal metal transition temperature versus $q = \tilde{E}_J/\tilde{E}_C = 4N^2\upsilon/\pi^2 E_J$. The solid line with no marks shows $\rho_s(q)$ in the absence of magnetic field including the fourth order corrections (equation (17)). The dashed line with no marks shows the same ρ_s but includes only the second order corrections. The solid line with crosses shows $\rho_s(q)$ in the presence of the magnetic frustration $f = 1/2$. Finally, the line with diamonds represents $\rho_s(\bar{q})$ in presence of strong random charge frustration. Note that in this last situation the relevant parameter is $\bar{q} = \tilde{E}_J^d/\tilde{E}_C = 4N^{3/2}\upsilon/\pi^2 E_J$. Dotted lines present just extrapolations of solid lines into the q range where corrections to ρ_s are not small. The circle and square marks on the bottom axes denote the points of the zero-temperature phase transitions (from the Table 1) for $f = 0$ and $f = 1/2$ respectively

temperature $T_{ins}(q)/T_{ins}(0)$. At $T > T_{\rm ins}$ Cooper pairs are unbound and array possesses nonzero thermally activated conductivity. Below $T_{\rm ins}$ linear conductivity vanishes (cf. [13, 14] for similar experimental observations in thin amorphous superconductive films).

Strong off-set charges: superconductor to "Coulomb glass" transition at $T = 0$. Now we concentrate on the case of strong random stray charges, but assume no real magnetic field present, $\gamma = 0$. Then dual "Josephson" couplings in the Hamiltonian (6) are diminished in magnitudes, so the parameter which controls quantum fluctuations is now

$$\bar{q} = \tilde{E}_J^d/\tilde{E}_C = 4N^{3/2}\upsilon/\pi^2 E_J, \tag{21}$$

and strongly frustrated by random phases, thus all effects related to vortex tunnelling are suppressed. In particular, it concerns reduction of superconducting density ρ_s due to vortex fluctuations, given (up to the 4-th order in $\tilde{E}_J$)

by

$$\rho_s = \frac{E_J}{N}\left(1 - \bar{q}^2 - 0.84\bar{q}^4\right) \tag{22}$$

In comparison with Eq.(17), note the absence of the ring diagram's contribution $a_r q^4$ which vanishes due to averaging over random phases (other terms contain magnitudes $|\Upsilon_{r,r'}|$ only). Eq.(22) provides reasonable accuracy up to $\bar{q} \approx 0.6$.

Upon sufficient increase of $\bar{q}$ the superconductive ground state will be destroyed. In the dual representation (6) it corresponds to formation at $\bar{q} = \bar{q}_c \sim 1$ of a gauge glass state (cf. e.g. [15]) with frozen in "vortex currents", *a la* persistent electric currents in magnetically frustrated random Josephson network. Physically it means an appearance of a collective insulating state with local lateral electric fields. At $\bar{q} \gg 1$ and $T = 0$ the corresponding "dual superfluid density" ρ_s^D scales as $\tilde{E}_J^d = 2\sqrt{N}\upsilon$. Gauge glass state in 2D nearest-neighbours array is unstable due to thermal fluctuations at any nonzero temperature [16], thus at any $T > 0$ our array will possess small but nonvanishing conductivity. The absence of finite-T charge unbinding transition demonstrates qualitative difference with the same model without random off-set charges, studied above.

Conclusions. We presented exact duality transformations for the JW array, proposed as a novel model system with superconductor-insulator QPT. Our main results are presented by Eqs.(17,18,22) for the array's macroscopic superconducting density ρ_s, and by Eq.(20) for dielectric permeability ε in the insulating state. Collective vortex oscillations with N-independent frequency (19) are predicted for the deeply insulating state in the model without off-set charges. In the opposite limit of strong charge disorder ω_J^D scales with N as $N^{-1/4}$. Variational estimates for QPT locations are presented in Eq.(15). $T \neq 0$ phase diagram is summarized in Fig.2. Low-temperature measurement of kinetic inductance seems to be the most adequate experimental method to study QPT in JW array. We are grateful to E. Cuevas, R. Fazio, L. B. Ioffe, S. E. Korshunov and M. Mueller for useful discussions. This research was supported by RFBR grant # 07-02-00310 and Programm "Quantum Macrophysics" of RAS. I.V.P. acknowledges support from Dynasty Foundation.

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